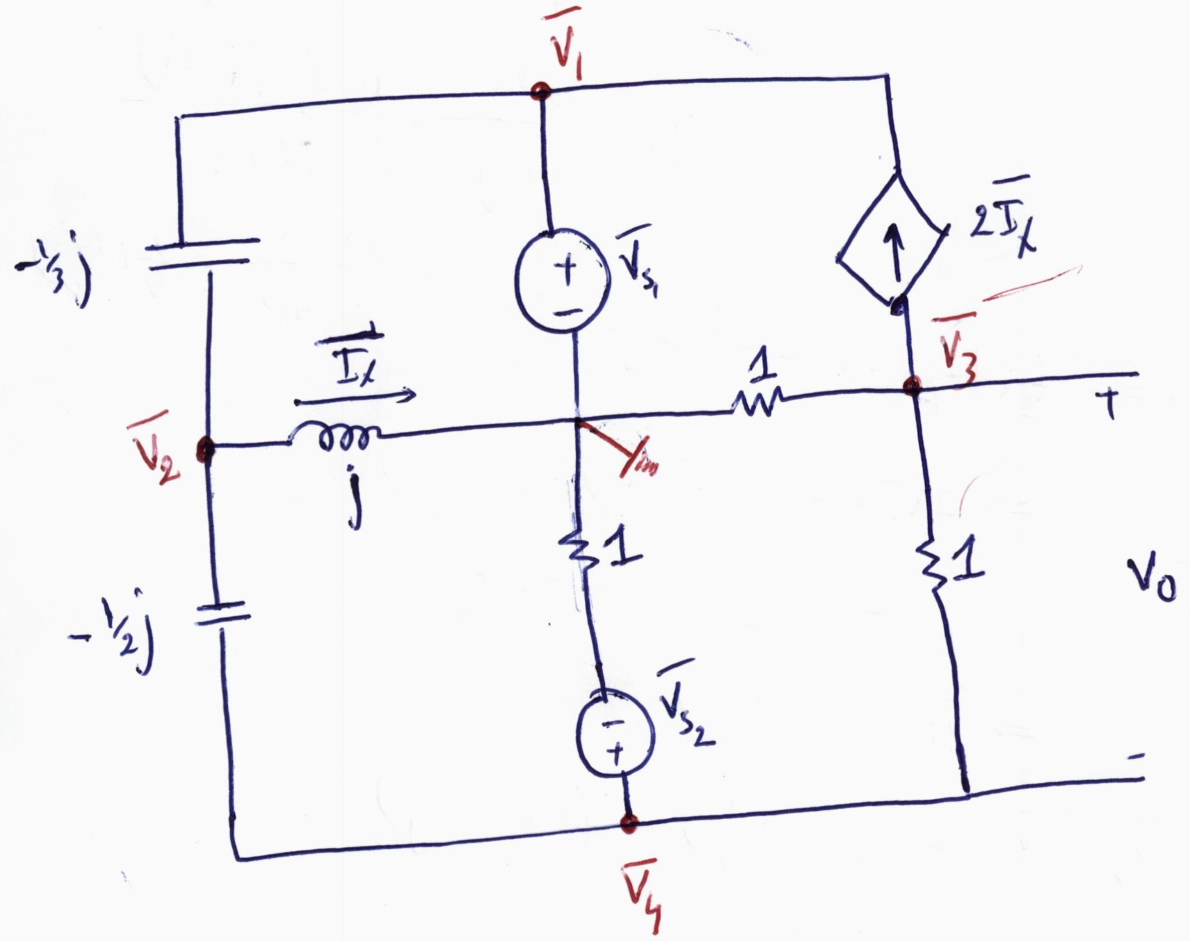
P3 Use Nodal analysis to Find $\overline{V_o}$.

$\overline{V_{s_1}} = \frac{2}{3}\angle 0 \quad , \quad \overline{V_{s_2}} = \sqrt{2}\angle -135$



node 1: $\overline{V_1} = \overline{V_{s_1}} = \frac{2}{3}\angle 0$

node 2: $\dfrac{\overline{V_2} - \overline{V_1}}{-\frac{1}{3}j} + \dfrac{\overline{V_2}}{j} + \dfrac{\overline{V_2} - \overline{V_4}}{-\frac{1}{2}j} = 0 \Rightarrow 4j\overline{V_2} - 2j\overline{V_4} = 2j$

$\boxed{\overline{V_4} = 2\overline{V_2} - 1}$ (1)

node 3: $\dfrac{\overline{V_3}}{1} + \dfrac{\overline{V_3} - \overline{V_4}}{1} + 2\overline{I_x} = 0$

$\overline{I_x} = \dfrac{\overline{V_2}}{j}$

$\Rightarrow 2\overline{V_3} - \overline{V_4} - 2j\overline{V_2} = 0$

$\boxed{2\overline{V_3} - 2(1+j)\overline{V_2} = -1}$ (2)

node 4: $\dfrac{\overline{V_4} - \overline{V_2}}{-\frac{1}{2}j} + \dfrac{\overline{V_4} - \overline{V_3}}{1} + \dfrac{\overline{V_4} - \overline{V_{s_2}}}{1} = 0 \Rightarrow (2+2j)\overline{V_4} - 2j\overline{V_2} - \overline{V_3} = -1-j$

$\boxed{(4+2j)\overline{V_2} - \overline{V_3} = 1+j}$ (3)

$$\left.\begin{aligned} 2\overline{V_3} - 2(1+j)\overline{V_2} &= -1 \\ (4+2j)\overline{V_2} - \overline{V_3} &= 1+j \end{aligned}\right\} \Rightarrow (8+4j-2-2j)\overline{V_2} = -1+2+2j$$

$$(6+2j)\overline{V_2} = 1+2j$$

$$\overline{V_2} = \frac{1+2j}{6+2j} = \frac{(1+2j)(6-2j)}{36+4}$$

$$= \frac{10+10j}{40} = \frac{1}{4}(1+j)$$

$$\overline{V_4} = 2\overline{V_2} - 1 = \frac{1}{2}(1+j) - 1 = -\frac{1}{2} + \frac{1}{2}j$$

$$2\overline{V_3} - 2(1+j)\overline{V_2} = -1 \Rightarrow 2\overline{V_3} = -1 + 2(1+j) \times \frac{1}{4}(1+j)$$

$$= -1 + \frac{1}{2} \times 2j = -1 + j$$

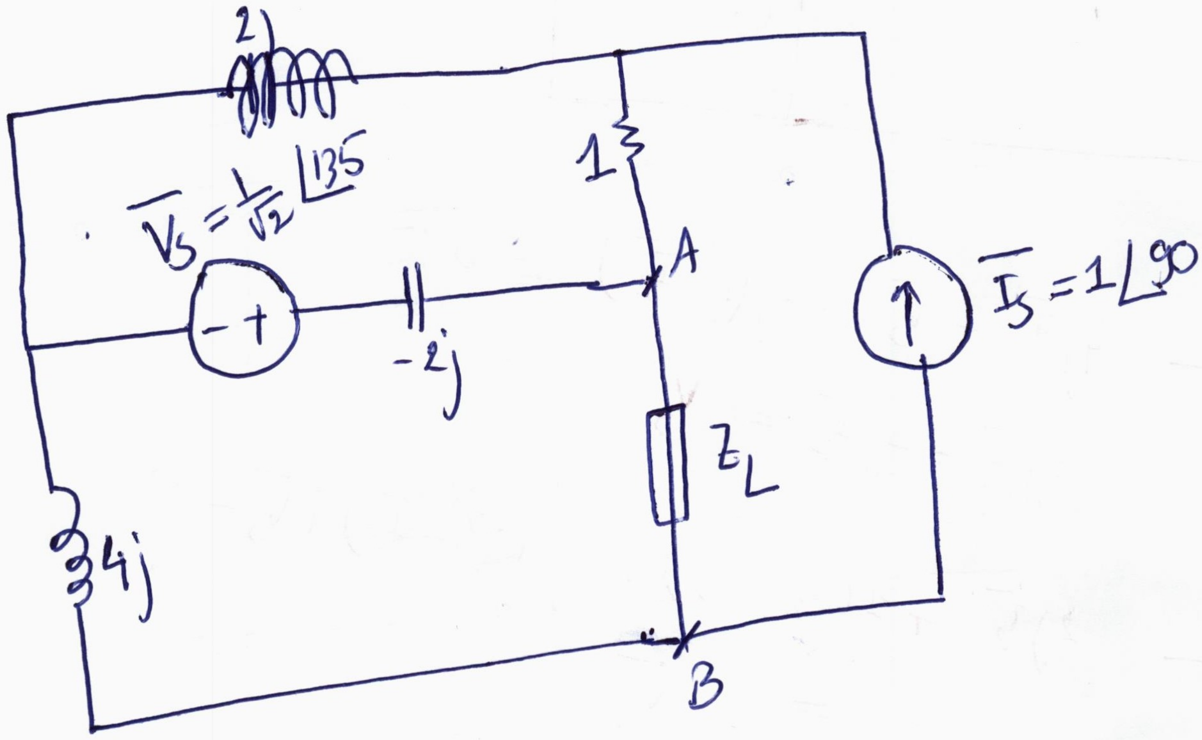
$$\Rightarrow \overline{V_3} = \frac{1}{2}(-1+j)$$

Finally $\overline{V_0} = \overline{V_3} - \overline{V_4} = \frac{1}{2}(-1+j) - \left(-\frac{1}{2} + \frac{1}{2}j\right)$

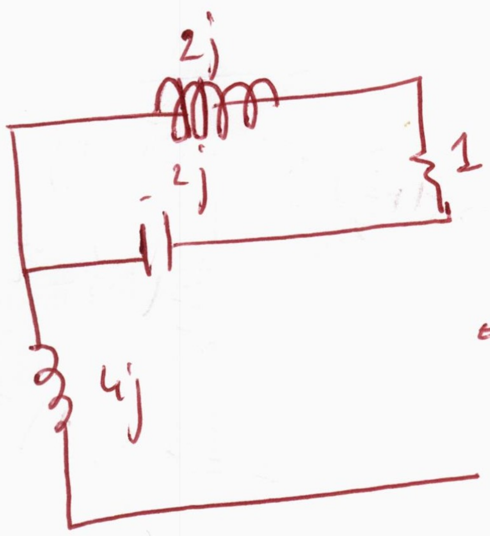
$$= 0 \text{ V}$$

P4: a) Find the Norton equivalent of the circuit between the nodes A and B as seen by the impedance Z_L

b) The Load Z_L consumes zero reactive power. Find the value of Z_L that would ensure a maximum power transfer

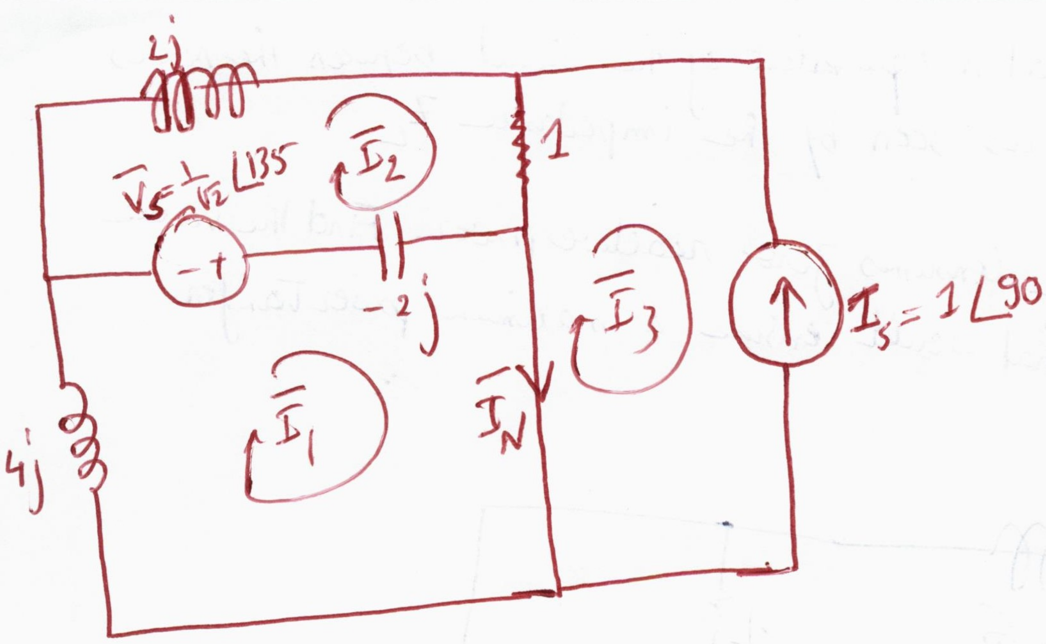


a)



$$Z_T = Z_{eq} = \frac{(1+2j) \times (-2j)}{1} + 4j$$

$$= 4 + 2j$$



Mesh 1: $4j\bar{I}_1 + \bar{V}_S - 2j(\bar{I}_1 - \bar{I}_2) = 0$

$2j\bar{I}_1 + 2j\bar{I}_2 = \bar{V}_S$

Mesh 2: $2j\bar{I}_2 + 1(\bar{I}_2 - \bar{I}_3) - 2j(\bar{I}_2 - \bar{I}_1) + \bar{V}_S = 0$

$\bar{I}_2 + 2j\bar{I}_1 - \bar{I}_3 = -\bar{V}_S$

Mesh 3: $\bar{I}_3 = -\bar{I}_S = -j$

Now

$$\begin{cases} 2j\bar{I}_1 + 2j\bar{I}_2 = \bar{V}_S \\ 2j\bar{I}_1 + \bar{I}_2 = -\bar{V}_S + \bar{I}_3 = -\bar{V}_S - j \end{cases} \Rightarrow (-1+2j)\bar{I}_2 = 2\bar{V}_S + j$$

$$= \frac{2}{\sqrt{2}}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j\right) + j$$

$$= -1 + j + j = -1 + 2j$$

$$\Rightarrow \bar{I}_2 = 1\angle 0$$

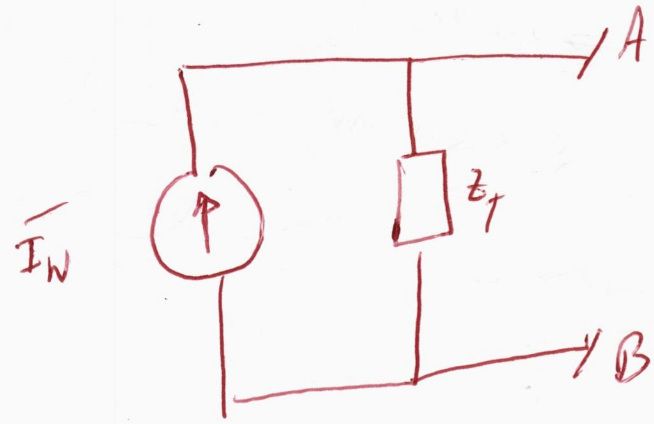
$$2j\bar{I}_1 + 2j\bar{I}_2 = \bar{V}_S \Rightarrow 2j\bar{I}_1 = \bar{V}_S - 2j\bar{I}_2 = \frac{1}{\sqrt{2}}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j\right) - 2j$$

$$= -\frac{1}{2} + \frac{1}{2}j - 2j = -\frac{1}{2} - \frac{3}{2}j$$

$\Rightarrow \boxed{\overline{I_1} = \frac{1}{4}j - \frac{3}{4}}$

Finally $\overline{I_N} = \overline{I_1} - \overline{I_3}$

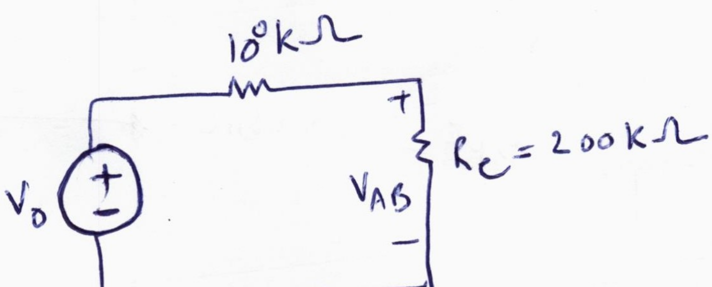
$$= \frac{1}{4}j - \frac{3}{4} - (-j) = -\frac{3}{4} + \frac{5}{4}j$$



b) Zero reactive power $\Rightarrow$ resistance

$$R_L = |Z_T| = \sqrt{16+4} = \sqrt{20}\ \Omega$$

P.1



$0 \leq V_0 \leq 10mV$

a. voltage divider: $V_{AB} = V_0 \times \frac{R_c}{R_c+R} = V_0 \times \frac{200}{300} = \frac{2}{3} V_0$

$$0 \leq V_{AB} \leq \frac{20}{3}\ mV$$

b. $\begin{cases} i_+ = i_- = 0 \\ V_+ = V_- \end{cases}$

Drop voltage on $R = 0 \Rightarrow V_+ = V_0 \Rightarrow V_- = V_0$

KCL @ $-$: $\dfrac{V_0}{R_1} + \dfrac{V_0 - V_s}{R_2} = 0 \Rightarrow \dfrac{V_0}{R_1} + \dfrac{V_0}{R_2} = \dfrac{V_s}{R_2}$

$\Rightarrow \dfrac{V_s}{V_0} = \left(1 + \dfrac{R_2}{R_1}\right) = 1 + 10 = 11$

c. add R_c KCL@ $-$: $\dfrac{V_0}{R_1} + \dfrac{V_0 - V_s}{R_2} = 0 \Rightarrow$

$\dfrac{V_s}{V_0} = 11 \Rightarrow V_s = 11 V_0$

$0 \leq V_s \leq 110 mV$

d. No drop voltage across $R = 100 k\Omega$

voltage gain

P_2: a. C_{eq}?

$(6 // 4) = 10$

$(2 // 3) = 5 \rightarrow 5 // 5$

$\rightarrow 10$ Series $10 = 5 \rightarrow 10$

$C_{eq} = 5 \mu F = 10$ Series 10

b. $W(t) = \frac{1}{2} C v^2(t)$

we will use $C_{eq} = 10 \mu F$

$$i(t) = C_{eq}\frac{dv(t)}{dt} \Rightarrow v(t) = \frac{1}{C_{eq}}\int_{t_0}^{t} i(t) + v(t_0)$$

- $0 < t \leq 2$: $i(t) = 5t$ mA,

$2 \leq t \leq 4$: $i(t) = 10$ mA,

$4 \leq t \leq 6$: $i(t) = (-10t + 50)$ mA,

Now, let us find $v(t)$ in (Volts)

$0 < t \leq 2$: $v(t) = \frac{1}{10}\int_0^t 5\tau \, d\tau + \cancel{v(0)}^{\,0}$

$= \frac{1}{10} \times 5 \times \frac{t^2}{2} = \frac{t^2}{4}$ (1)

$2 < t \leq 4$: $v(t) = \frac{1}{10}\int_2^t 10 \, d\tau + v(2)$

But $v(2) = \frac{2^2}{4} = 1$ V (From eq. (1))

$\Rightarrow v(t) = \frac{1}{10} \times 10 \times (t-2) + 1 = t - 1$ (2)

$4 < t \leq 6$: $v(t) = \frac{1}{10}\int_4^t (-10\tau + 50) \, d\tau + v(4)$

But $v(4) = 4 - 1 = 3$ V (From eq. (2))

$v(t) = \frac{1}{10}\left(-5\tau^2 + 50\tau\right)\Big|_4^t + 3$

$= \frac{1}{10}(-5t^2 + 50t + 5 \times 16 - 50 \times 4) + 3$

$= -\frac{1}{2}t^2 + 5t + 8 - 20 + 3 = -\frac{1}{2}t^2 + 5t - 9$ (3)

$$v(t) = \begin{cases} \frac{t^2}{4} & 0 \le t \le 2 \\ t-1 & 2 \le t \le 4 \\ -\frac{1}{2}t^2 + 5t - 9 & 4 \le t \le 6 \end{cases}$$

$$@\ t = 1.3 \Rightarrow \quad v(1.3) = \frac{(1.3)^2}{4} = 0.4225\,V \Rightarrow w(1.3) = \frac{1}{2} \times 10 \times (0.42)^2 = \boxed{0.89\,\mu J}$$

$$@\ t = 2.4 \Rightarrow \quad v(2.4) = 2.4 - 1 = 1.4\,V \Rightarrow w(2.4) = \frac{1}{2} \times 10 \times (1.4)^2 = \boxed{9.8\,\mu J}$$

$$@\ t = 5.5\,ms \Rightarrow v(5.5) = -\frac{1}{2}(5.5)^2 + 5 \times 5.5 - 9 = 3.375\,V$$

$$\Rightarrow w(5.5) = \frac{1}{2} \times 10 \times (3.375)^2 = \boxed{56.95\,\mu J}$$

A Second Method:

$$v(t) = \frac{1}{C_{eq}} \int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$$

$t_0 = 0$

$$= \frac{1}{C_{eq}} \int_0^t i(\tau)\,d\tau$$

$$v(1.3) = \frac{1}{10} \int_0^{1.3} i(\tau)\,d\tau = \frac{1}{10} \times \text{area under the curve of } i \text{ from } 0 \text{ to } 1.3$$

$$= \frac{1}{10} \times \frac{1.3 \times (1.3) \times 5}{2} = 0.4225\,V$$

$$v(2.4) = \frac{1}{10} \times \text{area under the curve from } 0 \text{ to } 2.4$$

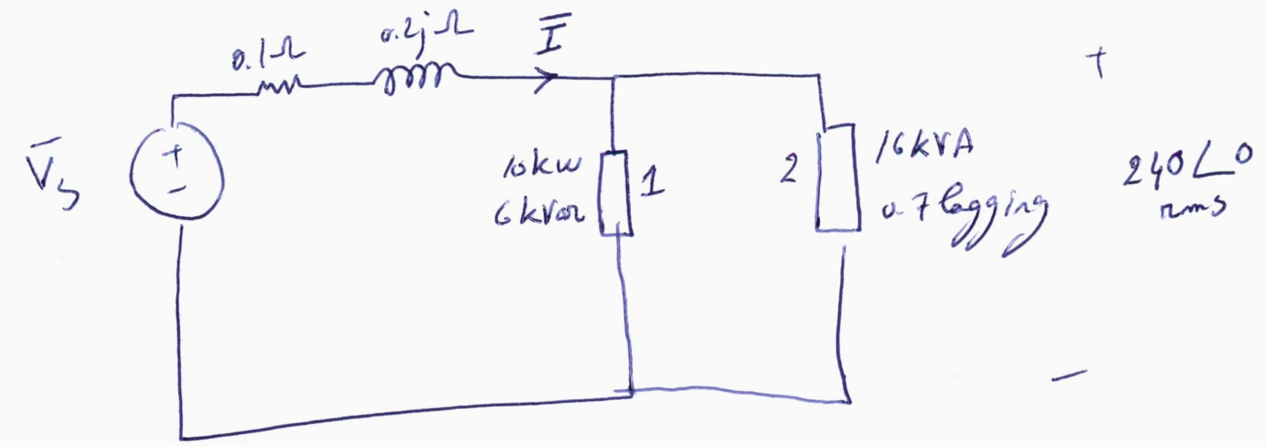
$$= \frac{1}{10}\left(\frac{2 \times 10}{2} + 0.4 \times 10\right) = 1 + 0.4 = 1.4\,V$$

$$v(5.5) = \frac{1}{10}\left(\frac{2 \times 10}{2} + 2 \times 10 + 1 \times \frac{10}{2} + \frac{0.5 \times (-5)}{2}\right)$$

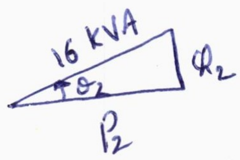
$$= 1 + 2 + 0.5 - 0.125 = 3.375\,V$$

Then Find $w(1.3)$, $w(2.4)$ and $w(5.5)$

Question 5:



a) power triangle for load 2:



$\cos\theta_2 = 0.7 \Rightarrow P_2 = 16 \times \cos\theta_2 = 16 \times 0.7 = 11.2$ KW

$Q_2 = +\sqrt{(16)^2 - (11.2)^2} = 11.43$ KVAR

As for load 1: it is given that $P_1 = 10$ KW and $Q_1 = 6$ KVAR

Thus the total power absorbed by loads 1 & 2 is $P_t = 11.2 + 10 = 21.2$ KW

the total reactive power of the loads 1 & 2 is $Q_t = 11.43 + 6 = 17.43$ KVAR

Hence $\tan\theta_t = \dfrac{Q_t}{P_t} = \dfrac{17.43}{21.2} = 0.822 \Rightarrow \boxed{\theta_t = 39.42^\circ}$

↳ this $\theta_V - \theta_I$ for the equivalent load of 1 & 2

To find I_{rms}, we have

$P_t = V_{rms} I_{rms} \cos\theta_t \Rightarrow I_{rms} = \dfrac{21.2 \times 10^3}{240 \times \cos(39.42)} = 114.34$ A

$\Rightarrow \bar{I} = I_{rms} \angle \theta_I = \boxed{114.34 \angle -39.42}_{rms}$

Since we have I: we find P_R and Q_L

$(Q_R=0)$ $(P_L=0)$

$P_R = R I_{rms}^2 = 0.1 \times (114.34)^2 = 1.307$ kW

$Q_L = X_L I_{rms}^2 = 0.2 \times (114.34)^2 = 2.614$ KVAR

Finally $P_{source} = P_t + P_R = 21.2 + 2.614 = 23.814$ kW

$Q_{source} = Q_t + Q_L = 17.43 + 2.614 = 20.044$ KVAR

b) $\cos\theta_2 = 0.9$ in this case. Using the same approach,

$P_2 = 16 \times \cos\theta_2 = 16 \times 0.9 = 14.4$ KW

$Q_2 = \sqrt{16^2 - (14.4)^2} = 6.97$ KVAR

$P_t = P_1 + P_2 = 10 + 14.4 = 24.4$ kW

$Q_t = Q_1 + Q_2 = 6 + 6.97 = 12.97$ kVAR

Hence $\tan\theta_t = \dfrac{12.97}{24.4} = 0.53 \Rightarrow \theta_t = 28° \Rightarrow \theta_V - \theta_I = 28 \Rightarrow$

$0 - \theta_I = 28 \Rightarrow \theta_I = -28°$

$P_t = V_{rms} I_{rms} \cos\theta_t \Rightarrow I_{rms} = \dfrac{24.4 \times 10^3}{240 \times \cos(28)} = 115.14$ A

$\Rightarrow I = 115.14 \angle -28$

$P_R = R I_{rms}^2 = 0.1 \times (115.14)^2 = 1.325$ kW

$Q_L = X_L I_{rms}^2 = 0.2 \times (115.14)^2 = 2.651$ KVAR

Finally $P_{source} = P_t + P_R = 24.4 + 1.325 = 25.725$ kW

$Q_{source} = Q_t + Q_L = 12.97 + 2.651 = 15.621$ KVAR

c) part a) The % loss of average power $= \frac{P_R}{P_{source}} \times 100 = \frac{1.307}{23.814} \times 100 = 5.488\%$

part b) The % loss of average power $= \frac{P_R}{P_{source}} \times 100 = \frac{1.325}{25.725} \times 100$

$= \underline{\underline{5.15\%}}$

$\hookrightarrow$ smaller loss

$\Longrightarrow$ Better